Utilization Elementary Siphons of Petri Net to Solved Deadloaks in Flexible Manufacturing Systems

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Abstract— This article presents an approach to the constructing a class structural analysis of Petri nets, where elementary siphons are mainly used in the development of a deadlock control policy of flexible manufacturing systems (FMSs), that has been exploited successfully for the design of supervisors of some supervisory control problems. Deadlock-free operation of FMSs is significant objectives of siphons in the Petri net. The structure analysis of Petri net models has efficiency in control of FMSs, however different policy can be implemented for the deadlock prevention. Petri nets models based deadlock prevention for FMS's has gained considerable interest in the development of control theory and methods for design, controlling, operation, and performance evaluation depending of the special class of Petri nets called S³PR. Both structural analysis and reachability tree analysis is used for the purposes analysis, simulation and control of Petri nets. In our experimental approach based to siphon is able to resolve the problem of deadlock occurred to Petri nets that are illustrated with an FMS.

Index Terms— deadlock prevention, simulation, elementary siphon, FMSs, Petri net, PN-Toolbox V. 2.3, S³PR

1. INTRODUCTION

A Flexible Assembly System (FAS) consists of a number of automated work stations connected with automated material handling equipment. FAS for mechanical parts should be designed to have the efficiency of a high volume robotic assembly line, where each robot repeats the same operation. Each robotic work station in FAS, though, is flexible and performs a wide variety of operations, enabling the system to simultaneously assemble for the different low volume parts types.

Flexible Assembly System and Flexible Manufacturing Systems (FMS's) at a broad level can both essentially be described as computer integrated manufacturing systems consisting of group of flexible automated work stations connected by automated material handling equipment. The industrial processes automation is always concerned with dynamic technical processes which are usually composed of asynchronously interfaced partial processes operating concurrently. The complexity of control systems designed increases considerably where some breakdowns (or deadlock) in the production system are (involving, e. g. tools, machines, automated guided vehicles (AGVs), robots, buffers and the material handling system), the flexibility in the volume of production, and variations in the rate of rejects are to be taken into account. To achieve their objectives, the system designers as well as the system operators have to use various system representations and various design techniques to achieve an outlook of the dynamic behaviour of a system, and to be able to improve this behaviour.

For this purpose, a powerful tool for modeling it is called Petri nets (PNs) can be used. Petri net is a modeling tool having the

ability of being applied for numerous of FAS and FMS.

A deadlock is the first defined in computer scientists when the time evolution of developing resource allocation logic in operating systems. In particular, Petri net is a content Deadlock property that is used to control systems to be deadlock-free. Parts are blocked waiting for resources in FMS hold by others that will never be granted. Many deadlock prevention approaches have been suggested in the literature [1,2, 3, 6] [8-10] and [13, 14, 17-19] for Petri net models of flexible manufacturing systems, based on siphon enumeration and control. In the deadlock **prevention** approach, some control is usually added to a system to ensure that the controlled system is live. In the deadlock **avoidance** approach [4, 5, 7], the control policy is generally used to determine (on-line) which system evolutions are correct. This approach usually leads to better use of resources, but, sometimes, it does not eliminate all deadlocks.

Petri net [12] is a well-known and important specification technique for concurrent and distributed systems. They allow a graphical representation of the system and its behaviour and there is a great variety of analysis techniques. These techniques can be divided into **static and dynamic** analysis.

Dynamic analysis techniques are based on the reachability graph, whose construction is rather time and space-consuming. The **reachability graph** of a Petri net dynamic analysis is a graphical representation of its possible firing sequences. Petri nets **a static analysis technique is** based on the method of invariants where is properties of a Petri net that hold in all markings and all the state of any systems. Static analysis is related to linear algebra and integer programming. An important feature of static technique is the structure of the net that is needed by static analysis techniques. It is independent of the initial marking and uses only structural net information, which is normally expressed by the incidence matrix of the net or represented the net structure. In the structural approach (static), it is required to identify certain parts of the Petri net, such as siphons, which has certain structural properties such as liveness, boundedness, or reversibility. Moreover, deadlock prevention is referring to **static resource allocation** policies to eliminate completely the possibility of deadlock occurrences.

Petri net is also a promising tool for describing and studying information processing systems [12] that are concurrent, asynchronous, and distributed systems. When PNs use of modeling on a real system, a powerful feature of Petri nets are their ability to represent good behavior properties of the system, such as liveness, (deadlock-freeness), boundedness, and reversibility. The liveness in FMS required ensures that deadlocks do not occur. When the number of raw components in FMS boundedness that is guarantees the buffer spaces and resources are bounded. Reversibility in the Petri nets that is enabling the system to return it's to initial state, thereby guaranteeing repetitive production. The competition for resources in an FMS may cause it to be deadlocked. of web services are equal to the existence of nonempty minimal siphons, which leads to the useful.

Petri nets have faced with considerable attention to information processing systems allow to model and visualize systems, which contain concurrence, resource sharing and synchronization. Applications of Petri nets are included computer communication networks, automated manufacturing systems (especially FMS), computer systems, workflow, communication protocols, web service, and software engineering verification. In [15, 16] the authors proposed a formal model for web service interaction with Petri nets. They detect that the compatibility method that the harmonization can be guaranteed by appending additional information channel's. Elementary siphons Petri net is also used in communication systems and Information Technology (IT) [16], such as composition of Web Services, and wireless sensor networks (WSN) are increasingly being required for applications where the data reliability needs to be duly guaranteed.

A special class of Petri nets, called a system of simple sequential processes with resources $(S^{3}PR)$ was an early work by Ezpeleta et al. [6], developed a design method of monitor-based liveness Petri net supervisors for FMS, which is usually considered to be a classical contribution that utilizes structural analysis techniques of Petri nets to prevent deadlocks in FMS. The deadlock prevention method can be achieved by adding a control place and related arcs to each strict minimal siphon (SMS), thus the SMS can never be emptied, which guarantees the liveness of the controlled system. There is proposed the relation-ship between strict minimal siphons and liveness of an S³PR is established. Elementary siphons were first proposed by Li et al. [8, 9] which are a subclass of SMSs. Apparently, the number is smaller the set of all SMSs. They used the method to control only elementary siphons and proved what the controlled system is live under some conditions.

siphon control policy that can obtain a small-sized supervisor with highly permissive behavior. They improve the policy of avoiding a complete, siphons enumeration [13]. The maximally permissive liveness-enforcing supervisors of computing, siphon for FMS can be presented in [13, 14]. They considered critical markings in a reachability graph algorithm to obtain maximally permissive controllers of iterations. They successfully identify the critical uncontrolled siphons and control them to make a deadlock-prone PN lives, although their algorithm also requires the repeated calculation of reachability graphs. Therefore, the policy not only solves the deadlock problem successfully, but also can be to obtain a maximally permissive controller.

Banaszak and Abdul-Hussin (2002) [5] developed a new approach for distributed deadlock avoidance control for Flexible Manufacturing Systems. They constructed feasible deadlock avoidance rules for exploiting the information about resource requirements of each particular operation of a manufacturing process and/or the repetitive character of the material and data processes to flow. They distributed control deadlock avoidance policies are "synchronization zones", a Request the Allocation Graph (RAG), and a capacity protocol allocation. A Request the Allocation Graph (RAG) concept of synchronization segments in a RAG model of a FMS operation. They proposed to assume that working processes requires access to a unit of a specific resource to execute particular technological operation. The efficiency of the procedure is higher then the efficiency of proceeding policies proposed in [4] and can be as high as the efficiency of the methods presented in [7].

Banaszak and Krogh (1990) [4] have developed an algorithm to avoid deadlock in FMS called Production Petri Nets (PPN). This has a class model a set of sequential processes sharing common resources. At each operation, only one resource is used and there are no alternative resource routings, which may lead to resource allocation conflict. Their Deadlock Avoidance Algorithm (DAA) is a restriction policy of constraining real-time resource allocation options. Deadlock is identified with being caused by circular wait relations between resources. The DAA is a feedback policy uses the current states of the resources and the known operation sequences in the active jobs to inhibit requests for resources only when they will potentially lead to deadlock condition. DAA partitions the production sequence in the process of subsequences or zones. Unshare "zones" to correspond to production steps using unshared resources. Shared zones correspond into production steps using shared resources. They proposed a deadlock avoidance algorithm [4] for a class of Petri net model formed into a set of sequential processes (without alternatives in its execution) that use a resource in each state. The algorithm controls the input of new tokens in a model zones, assuring that system evolution is always possible.

Organization. In section 2, provides the preliminaries to be used in this paper. The deadlock control policy is proposed, where is introduced the class ($S^{3}PR$) of Petri nets models, and the concept of elementary siphons in Section 3. Section 4, presents an FMS example to illustrate the applications of the proposed policy. Section 5, concludes papered.

The authors [13, 14] were developed a combined and selective

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2. PRELIMINAIRS

A Place/Transition-nets (P/T-net, for short), which represents the basic class in the sample family of Petri net models Murata [12].

Definition 1. A Petri net is a 4-tuple $\Psi = (P, T, E, W)$, where P, and T are finite, nonempty, and disjoint sets. P is a set of places, and T is a set of transitions with $P \cup T \neq \emptyset$, and $P \cap T = \emptyset$. $E \subseteq (P \ge T) \cup (T \ge P)$ is called a flow relation or the set of directed arcs. The net has represented by arcs with arrows from places to transitions or from transitions to places. W: $E \rightarrow Z^+$ is a mapping that assigns a weight to any arc, where $Z^+ = \{0, 1, 2, ...\}$. $\Psi = (P, T, E, W)$ is ordinary, denoted as $\Psi = (P, T, E, W)$, (When Weights W, of the arcs (W) = I, the net Ψ is called ordinary Petri net). The Weights W: $(P \ge T) \cup (T \ge P) \rightarrow Z^+$ is a mapping that assigns a weight to an arc: W(x, y) > 0 if $(x, y) \in E$, and W(x, y) = 0 otherwise, where $x, y \in P \cup T$ and Z^+ denote the set of non-negative integers.

Definition 2. In the absence of self-loops, an equivalent information is given by the incidence matrix. The computation of the incidence matrix C is subtracting C⁻ from C⁺ that is: C = (C⁺ – C⁻), or C_{ij} = post (t_j, p_i) – pre (p_i, t_j). A Petri net Ψ = (P, T, E, W) can be alternatively represented by its flow matrix or incidence C = (C_{ij}) which is defined by: C⁺(P x T) = W (E(P x T)) and C⁻ (P x T) = W(E(P x T)), also we can write C(p_i, t_j) = post(p_i, t_j) – pre(p_i, t_j), is the change in matrix C, where incidence matrix [C] of net Ψ is a $|P| \times |T|$ integer matrix and $[\Psi](p, t) = W(t, p) - W(p, t)$.

Definition 3. The preset of a node $x \in P \cup T$ is defined as $\bullet x = \{y \in P \cup T \mid (y, x) \in E\}$. While the post-set of a node $x \in P \cup T$ is defined as $x \bullet = \{y \in P \cup T \mid (x, y) \in E\}$. This notation can be extended to a set of nodes as: given $X \subseteq P \cup T$, $\bullet X = \bigcup_{x \in X}, \bullet x$ and $X \bullet = \bigcup_{x \in X}, x \bullet$. A marking is a mapping $M : P \rightarrow Z^+$, where $Z^+ \cup \{0\}$. M(p) denotes the number of tokens in place p. The preset (postset) of a set is defined as the union of the presets (postsets) of its elements. The pre- and post-sets of a transition t $\in T$ are defined respectively as: $\bullet t = \{p \mid Pre(p, t) > 0\}$, and $t \bullet = \{p \mid Post(p, t) > 0\}$. The pre- and post-sets of a place $p \in P$ are defined respectively as: $\bullet p = \{t \mid Post(p, t) > 0\}$ and $p \bullet = \{t \mid Pre(p, t) > 0\}$.

Definition 4. The pair (Ψ, M_0) is called a marked Petri net or a net system. The set of markings reachable from M in Ψ is denoted as $R(\Psi, M)$. A net (Ψ, M) is bounded iff $\exists k \in \Psi, \forall M \in R(\Psi, M_0), \forall p \in P, M(p) \le k$ holds. A transition $t \in T$ is enabled under M, denoted by M[t), iff $\forall p \in \bullet t, M(p) \ge 1$. A transition $t \in T$ is live under M_0 iff $\forall M \in R(\Psi, M_0), \exists M \in R(\Psi, M_0), M'[t)$ holds. (Ψ, M_0) is deadlock-free iff $\forall M \in R(\Psi, M_0), \exists t \in T, M[t)$ holds. (Ψ, M_0) is live iff $\forall t \in T, t$ is live under M_0 .

Definition 5. A transition t is said to be live if for any $M \in R(\Psi, M_0)$, there exists a sequence of transitions that is fired to lead next marking M' that enables at transition t. A PN is said to be live if all the transitions are live. A PN contains a deadlock if

there is a marking $M \in R(\Psi, M_0)$, at which no transition is enabled. Such a marking is called a dead marking. Deadlock situations are because of in appropriate resource allocation policies or exhaustive use of some or all resources. Liveness of a PN means that for each marking $M \in R(\Psi, M_0)$ reachable from the M_0 , it is finally possible to fire t, $\forall t \in T$ through some firing sequence. (Ψ, M_0) is said to be reversible, if $\forall M \in R(\Psi, M_0)$, $M_0 \in R(\Psi, M)$. Therefore, in a reversible net it is always possible to go back to initial marking M_0 . A marking M' is reachable from M. Reversibility is a special case of the home state property. If the home state $M' = M_0$, then the net is reversible.

Definition 6. A sequence of transitions $\sigma = \{t_1, t_2, \dots, t_n\}$ is a firing sequence of (Ψ, M_0) iff there exists a sequence of markings such that is: $M_0[t_1\rangle M_1[t_2\rangle M_2, \dots, [t_n\rangle M_n$. Moreover, marking M_n is said to be reachable from M_0 by firing σ , and this is denoted by $M_0[\sigma]M_n$. The firing sequence is a marking (M₁, M_2 , M_3 , ..., M_{n+1}) such that: ($\forall i, 1 \le i \le n$), and ($M_i[t_i \ M_{i+1})$, We can also write its by $[M_1[\sigma]M_{n+1}]$. The set of all markings reachable from M_0 is denoted by reachability set $R(M_0)$. The function $\sigma': T \rightarrow N^+$ is the firing count vector of the firable sequence σ , i.e. $\sigma'[t]$ represents the number of occurrences of $t \in T$ in σ . If $M_0[\sigma M]$, then we can write in vector form $M' = M_0 + C$. σ' , which is referred to as the linear state equation of the net. A marking M_0 is said to be potentially reachable iff $\exists X \ge 0$ such that: $M' = M_0 + C$. $\sigma \ge 0$, where σ is a firing sequence, a vector which is i-th denotes the number of occurrences of in σ . For $M_0[\sigma]M_n$, we have $M_n = M_0 + C \cdot \sigma_{t_i}$, which is called the state equation of net Ψ , where σ , called the firing count vector, is a vector which is i-th entry denotes the number of occurrences of t_i in σ.

Definition 7. A P-vector is a column vector I: $P \to Z$ indexed by P, where Z is the set of integers. I is a P-invariant (place invariant) if and only if (iff) $I \neq 0$ and $I^T \bullet [\Psi] = \mathbf{0}^T$ holds. P-invariant I is said to be a P-semiflow if every element of I is non-negative. $||I| ||^+ = \{p \in P \mid I(p) \neq 0\}$ is called the support of I. If I is a P-invariant of (Ψ, M_0) then $\forall M \in R(\Psi, M_0)$: $I^T \bullet M = I^T \bullet M_0$. In an ordinary net, siphon S is controlled by P-invariant I under M_0 if and only if $(I^T \bullet M_0 > 0)$ and $\{p \in P \mid I(p) > 0\} \subseteq S\}$. Such a siphon is called invariant-controlled siphons.

Definition 8. (Conservative). A Petri Net PN with the initial marking M_0 is strictly a conservative, if for all markings M, elements of the Reachability set $R(M_0)$, the number of tokens remains constants, i.e. A marked Petri net $\Psi = (\Psi, M_0)$, is said to be conservative iff :

 $\sum_{i=1}^{n} M(p_i) = \text{constant} \forall M \in R(M_0).$ If a marked Petri net is

conservative, then the sum of all tokens will remain a constant in all reachable markings.

$$\sum_{p_i \in p} M(p_i) = \sum_{p_i \in P} M_0(p_i).$$
 A PT-net $\Psi = (P, T, E, W)$ is said

to be conservative if and only if there exists a |P|-vector x > 0 such that x . C = 0, where C is the incidence matrix of Ψ , for a p-

invariant x and any markings M_i , $M_j \in \Psi^n$, which are reachable from M_0 by the firing of transitions, it holds. $X^T \cdot M = X^T \cdot M_0$.

Definition 9. Let S A non-empty sub-set of places. S \subseteq P is a siphon (trap) if and only if (iff) ${}^{\bullet}S \subseteq S^{\bullet}$ (resp. Q ${}^{\bullet} \subseteq {}^{\bullet}Q$). A siphon is said to be minimal iff there is no siphon contained in S it as a proper subset. A minimal siphon that does not contains the support of any P-invariant is called a strict minimal siphon (SMS). A siphon S is controlled iff it can never be emptied. It is said to be invariant-controlled by P-invariant I if $I^{T} \cdot M > 0$ and $||I||^{+} \subseteq S$.

Definition 10. A PN is live under M_0 iff $\forall t \in T$, t is live under M_0 . A transition $t \in T$ is live under M_0 iff $\forall M \in R(\Psi, M_0)$, $\exists M' \in R(\Psi, M_0)$, t is friable under M'. A transition $t \in T$ is dead under M_0 if $\exists M \in R(\Psi, M_0)$, where t is friable. A marking $M \in R(\Psi, M_0)$ is a (total) deadlock iff $\forall t \in T$, t is dead.

Example 1. The example bellows to illustrating a flexible assembly system. Petri net is models to describe a manufacturing system with a machine-tool and a robot that feeds with loading and unloading parts from/to a stock.

The flexible assembly system of Figure 1, consists of six circles, called places, used to represent six **conditions** labelled p_1 , p_2 , p_3 , p_4 , p_5 , and p_6 , and transitions, drawn as bars, representing three **events** labelled t_1 , t_2 , and t_3 . The directed arcs connect the places to the transitions and the transitions to the places. Each place contains zero or more tokens. A vector representation of the number of tokens over all places defines the marking of the Petri Net and hence the state of the system.

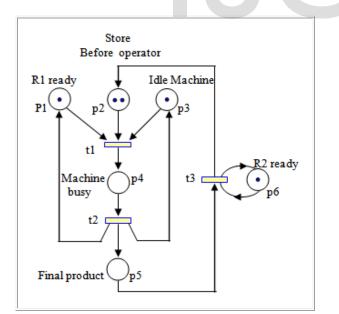


Figure 1: Petri net model of flexible assembly cell.

Robot1(R1) ready in place (p_1) , is transferring the raw material (or parts) from the storing in place (p_2) and puck up to the Machine-busy in place (p_4) . At the **some time** a machine-ready in place (p_3) inspection and the part is a suitable place for the opera-

tion place (p_4) of machine-busy. Transition t_1 fires to load machine-busy after completing the related operation by Machineready and put token in place p_4 . Machine-busy in place (p_4) is representing a condition that is an assembly operation in the process. Then transition t_2 is a fire the event occurs, that means to unload machine-busy putting final products in place (p_5) . Robot₂ in place (p_6) carries the final product of the store in place (p_2) . Then transition t_3 can be fired, removing a token from each of its input places (p_5) , and depositing a token is into placing (p_2) .

p 1	1	Robot1 ready	t ₁	Start loading
p ₂	2	Store with raw martial available	t ₂	Start unloading machine busy
p) 3	Machine ready	t ₃	Unloading final product
p 2	4	the assembly operation in process		
р) 5	Final product		
p) ₆	Robot2 Release final product		

Legend for the Petri net model of flexible assembly system.

We can assume that the **initial marking** is $M_0 = [1 \ 2 \ 1 \ 0 \ 0 \ 1]^T$. Transition t_1 is enabled and, when it fires, tokens are generated in places p_{4} , that is machine-busy. The new marking reached, $M_1 = [0 \ 1 \ 0 \ 1 \ 0 \ 1]^T$ can be expressed in the form.

Apply definition (6), state equation $M' = M_0 + C.\sigma_{ti}$.

The reachability set of the example 1, are give from the initial marking. When t_1 fires, the marking changes of the initial marking $M_0 = [1 \ 2 \ 1 \ 0 \ 0 \ 1]^T$ to $M_1 = [0 \ 1 \ 0 \ 1 \ 0 \ 1]^T$. Transition t_2 subsequently fires, the new marking are $M_2 = [1 \ 1 \ 1 \ 0 \ 1 \ 1]^T$. Firing t_3 return to the **initial marking**. The transition sequence (t_1, t_2, t_3) is a firing sequence of this Petri net. This firing sequence is represented by the firing vector. Fig. 1 show a P/T-net which is live, bounded, safe, reversible and conservative.

In the rest of this paper, in order to model a flexible assembly processes, we use the following interpretation of places, transitions and tokens:

- 1. Places represent operations or resources (i.e. machines, or robots).
- 2. If places represent the **resources**, tokens in it stand for a work-piece (raw part). Similarly, if places represent **operations**, a token in it represented the machines' operation being actually performed.
- 3. Transitions represent events reflecting changes of operations to that immediately following it or **resources** activity changes.

Example 2. In the Petri net shown in Fig. 2. The PN consist of five places and four transitions.

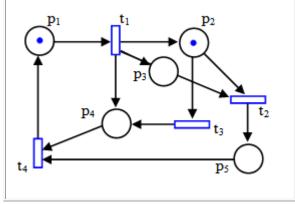


Fig. 2. A simple Petri net contain deadlock.

Let us consider an ordinary of a Petri net is shown in Fig. 2, which has three minimal siphons: $S_1 = \{p_1, p_2, p_5\}, S_2 = \{p_1, p_3, p_5\}$ p_5 , $S_3 = \{p_1, p_2, p_4\}$, and two minimal trap: $Q_1 = \{p_1, p_4\}$, and $Q_2 = \{p_1, p_3 p_5\}$. The analysis technical content the elemets of siphon is depending on definition 9. That is a siphon (trap) iff [•]S \subseteq S[•](resp. Q[•] \subseteq [•]Q). The set of places S₁ = {p₁, p₂, p₅}, constitutes a siphon, the set of pre-transitions is ${}^{\bullet}S_1 = \{t_4, t_1, t_2\}$, and the set of post-transitions is $S_1^{\bullet} = \{t_1, t_2, t_4\}$. Therefore, the set of siphon ${}^{\bullet}S_1 = [\{t_4, t_1, t_2\} \subseteq \{t_1, t_2, t_4\}]$ is true and ${}^{\bullet}S_1 \subseteq$ S_1^{\bullet} according definition 9. Another siphon analyzed is: $S_2 = \{p_1, p_2\}$ p_3, p_5 }, because ${}^{\bullet}S_2 = \{t_4, t_1, t_2\}$, and $S_2 {}^{\bullet} = \{t_1, t_2, t_4\}$, therefore, $[{}^{\bullet}S_2 = \{t_4, t_1, t_2\} \subseteq S_2^{\bullet} = \{t_1, t_2, t_4\}]$ is **true** depending on definition 9. So that, we can be complete the rest sets of siphons in this manner. Similarly, the calculation of a minimal trap is: Q_1 = {p₁, p₄}. The trap is: Q₁[•] ={t₁, t₄}, and [•]Q₁ ={t₄, t₁, t₃}, so that the subset of {t₁, t₄}[•] \subseteq [•]{t₄, t₁, t₃}, and [Q₁[•] ={t₁, t₄} $\subseteq {}^{\bullet}Q_1 = \{t_4, t_1, t_3\}$] is **true** depending on definition 9. Last a trap is: $Q_2 = \{p_1, p_3, p_5\}$. The set of post- transitions is contained in set of pre-transitions), i.e., $Q_2^{\bullet} = \{t_1, t_2, t_4\}$, and ${}^{\bullet}Q_2 = \{t_4, t_1, t_2\}$. So that the subset of $\{t_1, t_2, t_4\}^{\bullet} \subseteq \{t_4, t_1, t_2\}$, and Q_2^{\bullet} $\subset {}^{\bullet}Q_2$ is **true** depending on definition 9.

The Petri Net can be representation to an incidence matrix. The incidence matrix C of the net in Fig 3, is an (5 x 4) matrix. Petri net's of Fig. 2, has the following incidence matrices is shown in Fig 3. The Matrix $C = C^+ - C^-$.

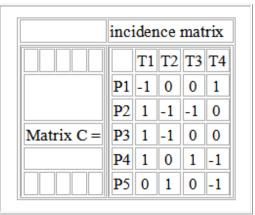


Fig. 3. The incidence matrix of Fig.2.

The analysis approach experiments in MATLAB for the study of

the so-called behavioural properties of Petri nets are the construction of the Coverability tree using the untimed Petri net is modelled with MATLAB [11] shown in Fig. 4.

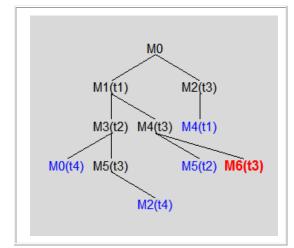


Fig. 4. Coverability tree of PN in MATLAB [11] of Fig. 2

The experimental results analysis of Petri net is shown in Fig. 2. The PN Toolbox in MATLAB is displaying the reachability tree as showing in Fig. 4. We can see that the deadlock is occurring at marking M6 when transition (t3) can not able to fired, having the red colored.

3. DEADLOCK ANALYSIS USING SIPHONS

In this paper, the deadlock analysis of general Petri nets is to be controlled by the class of an S³PR is considered. Deadlock prevention is considered to be a well-defined problem with resource allocation studies. It is usually achieved by using off-line computational mechanisms to control the request for resources to ensure that deadlocks never occur. We then show that these conditions can be expressed as a linear system of the initial marking. This provides a tool to analyze the structural deadlock-freeness of generalized Petri nets.

3.1. The Class of the S³PR Nets

The class of Petri nets investigated in this research is an $S^{3}PR$ that is first proposed in Ezpeleta et al. [6]. Before the presentation of its formal definitions is needed for our application. The following results are mainly from [6, 9].

Definition 11. A simple sequential process (S²P) is a Petri net $\Psi = (P_A \cup \{p^0\}, T, E)$, where the following statements are true: (1) $P_A \neq \emptyset$ is called a set of operation places:

- (1) $P_A \neq \emptyset$ is called a set of operation places; (2) $p^0 \notin P_A$ is called the process idle place;
- (3) A net Ψ is a strongly connected state machine; and
- (4) Every circuit of Ψ contains place p^0 .

Definition 12. Let $\Psi_i = (p_A \cup p_0 \cup p_R, T, E)$, be an S³PR. An initial marking M₀ is called an acceptable one if:

1) $\forall p \in p_0$, $M_0(p) \ge 1$; 2) $\forall p \in p_A$, $M_0(p) = 0$; and

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3) $\forall \mathbf{p} \in p_R$, $\mathbf{M}_0(\mathbf{p}) \ge 1$.

3.2. Elementary Siphon

The concepts elementary and dependent siphons are original work by Li et al. [8-10] and [17-19]. They are developed the Petri nets theory of computation and powerful mathematics. We have introduced the concept of elementary and dependent siphons as well as used in this paper.

Definition 13. Let $S \subseteq p$ is a subset of places of Petri net $\Psi = (P, T, F, W)$. P-vector λ_s is called the characteristic P-Vector of S if and only if $\forall p \in S$, $\lambda_s(p) = 1$; otherwise $\lambda_s(p) = 0$.

Definition 14. Let η_s is characteristic T-Vector of S if and only if $\eta_s^T = \lambda_s^T [\Psi]$.

Definition 15. Let $S \subseteq p$ is a subset of places of Petri net Ψ . η_s is characteristic T-Vector of S if and only if $\eta_s = \lambda_s^T .[C]$, where C is incidence matrix of the net Ψ .

Definition 16. Let $\Psi = (P, T, F, W)$ is a net with |P| = m, |T| = n, and K siphons $S_1 - S_k$, $k \in \Psi$. Let $\lambda_{s_i}(\eta_{s_i})$ is the characteristic P(T)-vector of siphon S_i , $i \in \Psi_k$. We define $[\lambda]_{k \times m} = [\lambda_{s_1} |\lambda_{s_2}| \dots |\lambda_{s_k}]^T$, and $[\eta]_{k \times n} = [\lambda]_{k \times m} \times [\Psi]_{m \times n} = [\eta_{s_1} |\eta_{s_2}| \dots |\eta_{s_k}]^T$. Where $[\lambda]([\eta])$ is called the characteristic P(T)-vector matrix of the siphons in net Ψ .

Definition 17. Let $\eta_{S_{\alpha}}, \eta_{S_{\beta}}, ..., \text{ and } \eta_{S_{\gamma}} = [\{ \alpha, \beta, ..., \gamma\} \subseteq \Psi_k \}$] a linearly independent maximal set of matrix [η]. Then $\Pi_E = \{S_{\alpha}, S_{\beta}, ..., S_{\gamma}\}$ is called a set of elementary siphons in net Ψ .

Definition 18. Let $S \notin \Pi_E$ be a siphon in Ψ net. Then S is called a strongly dependent siphon if $\eta_S =$

 $\sum_{S_i \in \Pi_E} a_i \eta_{S_i} \text{ holds, where } a_i \ge 0.$

Definition 19. Let $\{S_{\alpha}, S_{\beta}, ..., S_{\gamma}\}$ be the set of elementary siphons and $S \notin \{S_{\alpha}, S_{\beta}, ..., S_{\gamma}\}$ be a siphon in net Ψ . Then S is called a strict P redundant siphon if $\eta_{S} = \sum_{i \in \{\alpha, \beta, ..., \gamma\}} a_{i} \cdot \eta_{S_{i}}$ holds, where $a_{i} \ge 0$.

Theorem 1. The number of elements in any set of elementary siphons in net Ψ equals the rank of $[\eta]$.

Proof: Assume that Ψ has k siphons and k' elementary sphons. Clearly, we have $k \ge k'$. When there is no redundant siphon in a net, we have k = k'. Otherwise, there are k - k' redundant siphons in Ψ . By Definitions 17-19, η_{S_i} (i = k' + 1, k' + 2, ..., k) can be linearly represented by η_{S_j} (j = 1, 2, ..., k'). According to the definition of the rank of a matrix, we have that the rank of $[\eta]_{k\times n}$ is k'.

Theorem 2. Let Ψ_{ES} be the number of elementary siphons in $\Psi = (P, T, E, W)$. Then we have $\Psi_{ES} < \min \{|P|, |T|\}$. Proof: Let k be the number of siphons in Ψ net. Since $[\lambda]_{k \rtimes |P|} \times [\Psi]_{|P| \rtimes |T|} = [\eta]_{k \rtimes |T|}$, one can get $([\lambda]_{k \rtimes |P|}) + R([\Psi]_{|P| \rtimes |T|}) - P| \le R([\eta]_{k \rtimes |T|}) \le \min\{R([\lambda]_k \rtimes |P|), R([\Psi]_{|P| \rtimes |T|})\} \le R([\Psi]_{|P| \rtimes |T|})$. Thus, $\Psi_{ES} = R([\eta]_{k \rtimes |T|}) \le R([\Psi]_{|P| \rtimes |T|}) \le \min\{|P|, |T|\}$, which leads to the truth of this theorem.

4 AN S³PR FMS EXAMPLE

Let us consider the S³PR PN shown in Fig. 6, which have been studied in (e.g., [8], [10], [13] [17]). In this paper, we are a choice of different approaches to different the structure and initial markings in order to get different sets of siphons which have experimental results, and application to show the efficient methods the concept of elementary and dependent siphons in an S³PR net can be controlled of FMS.

We are considering an FMS [Fig. 5, (a) and (b)] with four machines M1, M2, M3 and M4 two robots R1 and R2. Each resource can hold a part at a time. Parts enter the cell through two loading buffers I1 and I2 and leave the cell through two unloading buffers O1 and O2. We consider two part P1, and P2. The production sequences of P1 and P2 are:

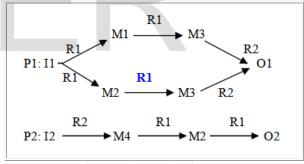


Fig. 5 (a) the production sequence

The FMS consists of four machines M1, M2, M3, and M4 (each can process only one part at a time), and two robots R1 and R2 (each can hold one part at a time) which can produce two parts-types.

The Petri nets model systems depicted in Fig. 6 is an S³PR that contains deadlock. We can see the original net system Ψ has total **282** reachable states, among which **16** are deadlocks state.

A Petri net is based deadlock prevention policy of (FMS) through a special class of Petri Nets that we call S³PR. We can apply definition 11, which are representing an (S³PR). There are 19 places and 14 transitions. A Petri nets based deadlock prevention policy of (FMS) through a special class of Petri Nets that we call S³PR. The places have the following set partition: $P_0 = \{p_1, p_7\}$, $P_R = (p_5, p_9, p_{12}, p_{17}, p_{18}, p_{19})$, $P = P^2 \cup P^1 = \{p_{11}, p_{13}, p_{14}, p_{15}, p_{16}\} \cup \{p_2, p_3, p_4, p_6, p_8, p_{10}\} = \{p_2, p_3, p_4, p_6, p_8, p_{10}, p_{11}, p_{13}, p_{14}, p_{15}, p_{16}\}$. The PN $\Psi = (\Psi, M_0)$ is an S³PR.

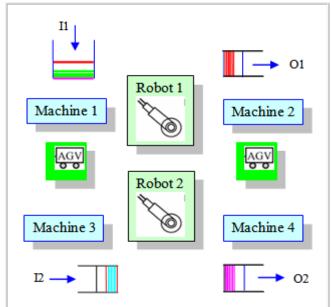
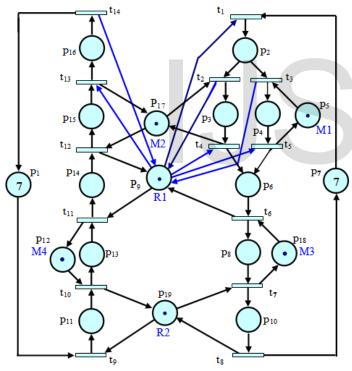
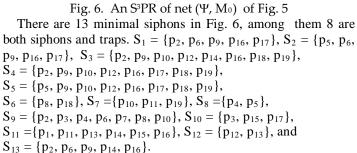


Fig. 5 (b) the layout of an FMS with four machines





The set of siphons from S_6-S_{13} , is both a siphon and trap. We are choices strict minimal siphons S_1 , S_2 , S_3 , S_4 , and S_5 which is

not contain any traps. We can apply the concept the elementary and dependent siphon is firstly proposed by Li and Zhou [8, 9]. The characteristic P/T- vector matrix $(|\lambda|, |\eta|)$ is as follows:

$$\begin{split} \lambda S_1 &= (0,1,0,0,0,1,0,0,1,0,0,0,0,0,0,1,1,0,0)^T, \\ \lambda S_2 &= (0,0,0,0,1,1,0,0,1,0,0,0,0,0,0,1,1,0,0)^T, \\ \lambda S_3 &= (0,1,0,0,0,0,0,0,1,1,0,1,0,1,0,1,0,1,1)^T, \\ \lambda S_5 &= (0,1,0,0,0,0,0,0,1,1,0,1,0,0,0,1,1,1,1)^T. \\ \lambda S_4 &= (0,0,0,0,1,0,0,0,1,1,0,1,0,0,0,1,1,1,1)^T. \end{split}$$

The T-vectors are structure as follow:

$$\begin{split} \eta S_1 &= -t_1 + t_3 + t_4 - t_9 + t_{13} \\ \eta S_2 &= -t_1 + t_4 + t_5 - t_9 + t_{13} \\ \eta S_3 &= -t_1 + t_7 - t_9 + t_{11} \\ \eta S_4 &= -2t_1 + t_4 + t_5 + t_7 - 2t_9 + t_{11} + t_{13} \\ \eta S_5 &= -2t_1 + t_3 + t_4 + t_7 - 2t_9 + t_{11} + t_{13} \end{split}$$

In addition, the linearly independent T-vectors can be constructed in $[\eta]$ shown as follows:

t	I	<u>t</u> 1	<u>t</u> 2	<u>t</u> 3	<u>t</u> 4	<u>t</u> 5	<u>t</u> 6	<u>t</u> 7	<u>t</u> 8	<u>t</u> o	<u>t₁₀</u>	<u>t</u> 11	<u>t12</u>	<u>t₁₃</u>	t ₁₄	l
η _{\$1} =	I	-1	0	1	1	0	0	0	0	-1	0	0	0	1	0	I
η _{\$2} =	I	-1	0	0	1	1	0	0	0	-1	0	0	0	1	0	I
η _{\$3} =	I	-1	0	0	0	0	0	1	0	-1	0	1	0	0	0	I
η _{\$4} =	I	-2	0	0	1	1	0	1	0	-2	0	1	0	1	0	+
η ₈₅ =	I	-2	0	1	1	0	0	1	0	-2	0	1	0	1	0	+

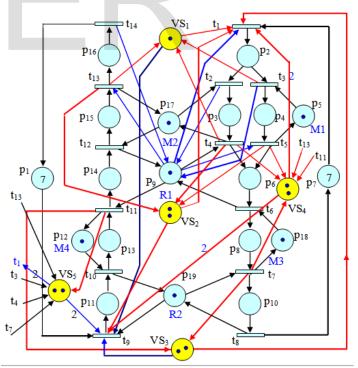


Fig. 7. The supervisor (Ψ_1, M_1) of plant net model of (Ψ, M_0) Where $\eta S_4 = \eta S_2 + \eta S_3$, and $\eta S_5 = \eta S_1 + \eta S_3$ is a T-vector. If are the elementary siphon identification algorithm, proposed in [9], $\Pi_E = (S_1, S_2, S_3)$ is a set of elementary siphons and S_4 and S_5 are **dependent** ones. According to the results develo

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In order to control a system of such a plant model is depicted in Fig. 6. We can be obtained firstly by adding three controls place (monitors) to the plant Petri net model for constraint the behavior of the modelled system such that deadlocks can never occur. A siphon is generating even though the monitors are added. The reachability graph has **57** states. The three control places are represented elementary siphons. A siphon {S₁, S₂, S₃} in Fig. 7, are present as the set of elementary siphons, and the control places VS₁–VS₃ are added to make S₁, S₂ and S₃ invariant-controlled, respectively.

Note that, when control places are added of $VS_1 - VS_5$, we can see that the reachability graph has 18 states, and the resultant net is also deadlock-free as shown in Fig. 8, used Petri net tool with MATLAB tool [11].

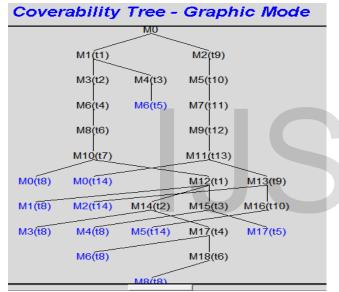


Fig. 8. Reachability tree is deadlock-free in MATLAB of Fig. 7.

Let us find out a P-invariant of the Petri net is a model in Fig. 2. According to definition 7, there are three minimal siphons, which can be emptied. There is $S_1 = \{p_2, p_6, p_9, p_{16}, p_{17}\}$, $S_2 = \{p_5, p_6, p_9, p_{16}, p_{17}\}$, and $S_3 = \{p_2, p_9, p_{10}, p_{12}, p_{14}, p_{16}, p_{18}, P_{19}\}$. The control place VS₁ is added according to P-invariant such: $I_1 = (0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1VS_1)$ is a P-invariant of the resultant net (Ψ_1 , M_1). Therefore,

 $I_1 \cdot M_1 = 0$ by definition 7, and it is easy to compute that: [(Ψ_1](VS₁, t) = $-t_1 + t_3 + t_4 - t_9 + t_{13}$. Similarly,

$$\begin{split} I_3 &= (0, 1, 1, 1, 0, 1, 0, 1, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1VS_3) \text{ is} \\ \text{also a P-invariant of } (\Psi_1, M_1). \text{ Therefore, } (I_3 \cdot M_1) &= 0, \text{ and} \\ [\Psi_1] (VS_3, t) &= -t_1 + t_7 - t_9 + t_{11}. \text{ Where } M_1(VS_1) &= M_0(S_1) - 1 \\ &= 1, M_1(VS_2) &= M_0(S_2) - 1 &= 2, \text{ and } M_1 (VS_3) &= M_0(S_3) - 1 &= 2. \\ \text{After adding } VS_1, VS_2 \text{ and } VS_3 \text{ to } (\Psi, M_0), \text{ the new model } (\Psi_1, M_1) \text{ is deadlock-free (actually live), and the reachability graph} \end{split}$$

has **57** states.

For example, the Fig. 7 is to show an $S^{3}PR$ net.

 $S_1 = \{p_2, p_6, p_9, p_{16}, p_{17}\}$ is a minimal siphon of the net. We can apply **definition 7**, to find a P- invariant of the net depending on the incidence matrix of Fig. 7. So that, siphon S_1 is controlled by P-invariant:

$$\begin{split} I_{S1} &= (0, 0, 0, 0, 0, 1, 0, 0, 1, 0, -1, 0, -1, 0, 0, 1, 1, 0, 0, -1VS_1)^T, \\ I_{S1} &= (p_2 + p_6 + p_9 + p_{16} + p_{17} - p_2 - p_{11} - p_{13} - 1VS_1) \text{ is a } P - \\ \text{invariant of } (\Psi_1, M_1). \text{ Where, } I^T \cdot M_0 &= \{M(p_2) + M(p_6) + M(p_9) + M(p_{16}) - M(p_{17}) - M(p_2) - M(p_{11}) - M(p_{13}) - M(VS_1)\} = 1 > \\ 0. \text{ So that } \|I_1\|^+ &= \{p_2, p_6, p_9, p_{16}, p_{17}\} \subseteq S. \text{ Thus, } S_1 \text{ is an invariant-controlled siphon and it can never be emptiable. Similarly, for } S_2 = \{p_5, p_6, p_9, p_{16}, p_{17}\}, \text{ is a siphon of the net. Siphon } S_2 \text{ is controlled by } P - \text{invariant by:} \end{split}$$

$$\begin{split} &I_2 = (0, 0, 0, 0, 1, 1, 0, 0, 1, 0, -1, 0, -1, 0, 0, 1, 1, 0, 0, \\ &-1VS_2)^T, \ I_{S2} = (p_5 + p_6 + p_9 + p_{16} + p_{17} - p_{11} - p_{13} - 1VS_2), \text{ is a} \\ &P-\text{invariant of } (\Psi_1, M_1). \text{ Where, } I^T \bullet M_0 = \{M(p_5) + M(p_6) + \\ &M(p_9) + M(p_{16}) + M(p_{17}) - M(p_{11}) - M(p_{13}) - M(1VS_2)\} = 1 > \\ &0. \text{ So that } \|I_2\|^+ = \{p_5, p_6, p_9, p_{16}, p_{17}\} \subseteq S, \text{ is an invariant-controlled siphon and it can never be emptied. Similarly, } S_3 = \\ &\{p_2, p_9, p_{10}, p_{12}, p_{14}, p_{16}, p_{18}, p_{19}\}, \text{ is a siphon of the net. A P-invariant, control to siphon } S_3 \text{ by: } I_3 = (0, 0, -1, -1, 0, 0, 0, 0, 1, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, -1VS_3)^T, \text{ where } I_{S3} = (p_2 + p_9 + p_{10} + p_{12} + p_{14} + p_{16} + p_{18} + p_{19} - p_2 - p_3 - p_4 - 1VS_3) \text{ is a P-invariant of } \\ &(\Psi_1, M_1). \text{ Where, } I_3 \bullet M_0 = \{M(p_2) + M(p_9) + M(p_{10}) + M(p_{12}) + M(p_{14}) + M(p_{16}) + M(p_{18}) + M(p_{19}) - M(p_2) - M(p_3) - M(p_4) \\ &- M(1VS_3)\} = 1 > 0. \\ \end{split}$$

 $||I_3||^+ = S_3 \subseteq S$. Thus, S_3 is an invariant-controlled siphon and it can never be emptiable respectively.

Controlling the three essential siphons with the P-invariant based method results of the additional places $VS_1 - VS_3$ is elementary siphons. In order to reduce the reachable graph, we can add $VS_1 - VS_5$. That is verifying the controllability of dependent siphons S_4 and S_5 . Note that $\eta S_4 = \eta S_2 + \eta S_3$ and $\eta S_5 = \eta S_1 + \eta S_3$, which indicate that S_4 and S_5 are strongly dependent, and the reachability graph has **18 states** as shown in Fig. 7. The resulting controlled Petri net is shown in Fig. 8, the coverability tree PN tool with MATLAB [11]. It is being verified that dead-lock does not occur to this Petri net.

5 CONCLUSION

The deadlock prevention policy developed in this paper is based on the structural analysis of Petri nets. We can distinguish to siphon in a Petri net model of elementary and dependent ones. A monitor is added to the plant model such as the siphon is Pinvariant-controlled for each elementary siphon, where integerprogramming technique is used to guarantee that no emptiable control-induced siphon is generated. The main advantages of Petri nets structure are properties that have been exploited successfully via the design of supervisors of some supervisory control problems. This paper has specifics on the existence of marking S³PR based on the concept of elementary siphons that play an important role in the design of structurally simple livenessenforcing net supervisors.

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